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AUTHOR Romberg, Thomas A.
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ABSTRACT

Data on how many minutes of instruction were allocated to various aspects of teaching initial addition and subtraction concepts and skills in the Developing Mathematical Processes (DMP) curriculum were summarized for 20 classrooms in grades 1-3. The same curriculum materials were used in each class at each grade level. The number of minutes spent on the 148 specific parts of the curriculum were observed. Each part was then classified in terms of 29 variables (40 codes). From this, data summaries of time spent on each code were prepared for each class. The summary data revealed that each class varied from others in important ways. However, four important features were apparent: (1) classes differed more on total allocated time than in terms of any other characteristic; (2) modification of the curriculum was generally made in all classes to stress practice and skill acquisition and to reduce the time spent on exploration and discussion of mathematical ideas; (3) if students were judged to be "poor," then even more practice and less exploration were given; (4) if students were judged to be "good," then in addition they were given opportunity to explore and discuss ideas. (Author/MNS)

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Wisconsin Center for Education Research
University of Wisconsin-Madison · School of Education
1025 West Johnson Street · Madison, Wisconsin 53706 · (608) 263-4200

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Allocated Time and Content Covered in Mathematics Classrooms

Thomas A. Romberg

Professor

University of Wisconsin-Madison

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Abstract

In this report the quantitative data on how many minutes of instruction were allocated to various aspects of a curriculum designed to teach initial addition and subtraction concepts and skills is summarized. That data was collected at grades 1, 2, and 3 in 8, 6, and 6 classrooms per grade respectively. The same curriculum materials were used in each class at each grade level. The number of minutes spent on the 148 specific parts of the curriculum were observed. Each part was then classified in terms of 29 variables (40 codes). From this, data summaries of time spent on each code were prepared for each class.

The summary data reveals that each class varies from others in important ways. However, four important features are apparent:

1. Classes differ more on total allocated time than in terms of any other characteristic.

2. Modification of the curriculum programs are generally made in all classes to stress practice and skill acquisition and to reduce the time spent on exploration and discussion of mathematical ideas.

3. If students were judged to be "poor," then even more practice and less exploration was emphasized.

4. If students were judged to be "good," then in addition they were given the opportunity to explore and discuss ideas.

Allocated Time and Content Covered in Mathematics Classrooms

Anyone who has observed the same mathematical content being taught in different elementary classrooms has seen that each classroom seems to operate differently from the others. Sometimes the differences are striking and other times they are subtle. Sources for these differences are many: different curricular materials, children with different backgrounds, teacher personality differences, different emphasis on aspects of a "hidden curriculum," and so forth. The cumulative effects on pupil performance of these operational differences is not always clear even though one is sure some classes are "better" or more "exciting" than others.

The purpose of this paper is to summarize the observed quantitative differences in content inclusion and emphasis in several mathematics classrooms which we believe influence what students learn.[1] Documenting differences in content coverage in mathematics classrooms was one facet of the three-year longitudinal study conducted by the Mathematics Work Group of the R & D Center (Romberg, Carpenter, & Moser, 1978).

Population

The population recruited in 1978 was eight first-grade classrooms in three different schools (containing two, three, and three classes, respectively). The three schools were in the local Madison area so that data could be easily gathered. All three serve a middle class population, are organized around self-contained, age-graded classes, and all had previously used Developing Mathematical Processes (DMP) (Romberg, Harvey, Moser, & Montgomery, 1974, 75, 76) as their instructional program in mathematics.

During the first year of the study, approximately 150 children were involved. At the end of the first year, one school elected not to participate further in the study. As a result, the number of children was reduced to about 100 for the last two years. By the completion of the longitudinal study, in spring 1981, 20 teachers had been involved.

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[1] A more complete description appears in Romberg, Stephens, Buchanan, and Steinberg, 1983.

Curriculum Materials

For this study the learning tasks were clearly defined. Addition and Subtraction was the content area in mathematics. All instruction was based on the same curricular materials, which were specially prepared for the study. Ten curriculum units were written in the general pedagogic style of DMP with which the teachers were familiar (see Table 1). Six were designed to teach the representation skills associated with addition and subtraction problem situations, together with the basic addition and subtraction facts (Kouba & Moser, 1979). The other four were designed to teach algorithmic skills (Kouba & Moser, 1980). Each curriculum unit was designed to take two to four weeks to teach. The first three units dealing with representation were taught in the spring semester 1978-79 (grade 1). The remaining three on representation and the first two on algorithms were taught in 1979-80 (grade 2). The last two units on algorithms were taught in the fall semester 1980-81 (grade 3).

Furthermore, each of the 10 topics is made up of several activities and each activity is separated into one or more parts (see Table 1). The basic instructional unit is an activity part. In total, there are 148 parts, 67 activities, and 10 topics. All information was gathered with respect to each part.

Content Coverage Variables

A scheme for classifying content characteristics was developed. Each activity was coded in terms of 40 codes representing 29 variables. Hence, a data base exists of 5290 bits of information about instruction on curriculum parts (40 codes x 148 parts). Combining this with the observational data, we know the number of minutes actually spent on each "bit" of content in each classroom. These codes include 21 on content goals, 18 on specific content objectives, and the appropriate time estimated for each part.

Content goal variables. First, each part of each activity of the 10 topics S1-S6, A1-A4 (see Table 1) was classified according to the objectives stated by the DMP authors.

Variable 1: DMP objectives. Four codes were derived from teachers' notes in the text material. Three classifications used were: preparatory, regular, and review. A fourth classification was devised to indicate the presence of preparatory and regular objectives in the same part.

Table 1

Instructional Topics, Number of Activities and Parts, and Recommended Time for the Longitudinal Study on Addition and Subtraction

Area	Topic	Title	Number of Activities	Number of Parts	Recommended Time
Sentence Writing					
Grade 1	S1	Initial Sentence Writing	7	13	505
	S2	Sentence Writing: Part-Whole and Difference	8	15	665
	S3	Solving Number Sentences	6	12	505
Grade 2	S4	Solving Situations 0-20	7	12	625
	S5	Solving Situations and Sentences 0-20	8	15	615
	S6	Mastering Writing and Solving Sentences 0-20	9	26	900
Algorithms					
	A1	Addition/Subrraction 0-99 No Regrouping	5	13	490
	A2	Addition of Two-digit Numbers with Regrouping	7	14	550
Grade 3	A3	Subtraction of Two-digit Numbers with Regrouping	5	13	540
	A4	Solving: The Numbers 0-99	6	15	485
Totals	10 Topics		69 Activities	148 Parts	5880 Minutes

Quality of Content Variables. Second, each part was coded in terms of nine "quality-of-content" goal variables. These quality-of-content goals were first discussed by Marian Small in Research on Teaching from a Curricular Perspective (Romberg, Small, & Carnahan, 1979).

Variable 2: Importance of objectives. The importance of objectives was assessed as being of great, moderate, or little importance. The latter category was reserved only for those units presented as optional to the teacher.

Variable 3: Algorithmic nature of teaching. This goal was judged to be present if a part of an activity involved the acquisition or practice of a skill. It was judged to be absent if the part involved sentence writing, problem solving or some other non-algorithmic activity. Some parts of materials embodied both goals.

Variable 4: Intended outcomes were derived from the teachers' notes. These outcomes were (a) maintenance of concepts, skills, etc., (b) acquisition of new concepts, skills, (c) preparation for new concepts, skills to be introduced in subsequent activities.

Variable 5: Concreteness was considered to be a feature if opportunities were provided in the part for the use of manipulatives of any sort (e.g., counters, blocks, apparatus). A part was considered to lack concreteness if these opportunities were not present.

Variable 6: Diversity of approach was considered to be a feature if the part departed from the "standard" approach of the units. The standard approach involved the use of algorithms or sentence writing using whole number quantities. A diverse, or non-standard, approach did not incorporate numbers quantified or use of algorithms, e.g., use of lengths instead of number quantities, or pencils in pots. Two classifications were used: diverse (non-standard) and not diverse (standard).

Variable 7: Applications were judged to be absent if the part involved writing number sentences or using algorithms apart from relating the sentence or algorithm to a verbal problem. Standard applications (S) were present if students were expected to deal with "real" situations by means of verbal problems. Other applications (O) were indicated through the use of money, graphs, measurement, dice, etc.

Variable 8: Explorability was judged by the inclusion of activities which did not have predetermined answers. Explorability was considered to be absent if activities presented to students had predetermined answers.

Variable 9: Encouragement of pupil discussion was judged to be present if the part made specific provision for pupil/pupil or pupil/teacher discussion. If specific provisions were not made, then this feature was considered to be absent.

Variable 10: Cohesion. A part was coded as being cohesive if the teacher, in addition to introducing and developing an activity, was asked to summarize or discuss children's results. This feature was not limited to teachers' remarks at the conclusion of a part, but was judged to be present if the teacher is encouraged to make summarizing remarks about or to discuss the outcomes of children's work in that part.

Content objective variables. The content of the topics was also classified in terms of the 18 intended instructional objectives under three general headings.

Sentence writing objectives include those situations where the student is given a verbal problem, involving the numbers 0-20 (or 0-99) that is solvable by using either addition or subtraction, and the pupil is to write a sentence representing that situation. These objectives are specified for seven types of verbal problems; and for each type a further distinction was made according to whether the numbers 0-20 or 0-99 were required either for writing the sentence or for its solution. These 14 variables are:

Variable 11. Simple joining (addition) (0-20)
Variable 12. Simple joining (addition) (0-99)
Variable 13. Simple separating (subtraction) (0-20)
Variable 14. Simple separating (subtraction) (0-99)
Variable 15. Part-part-whole (subtraction) (0-20)
Variable 16. Part-part-whole (subtraction (0-99)
Variable 17. Part-part-whole (addition) (0-20)
Variable 18. Part-part-whole (addition) (0-99)
Variable 19. Comparison (subtraction) (0-20)
Variable 20. Comparison (subtraction) (0-99)
Variable 21. Joining-missing addend (subtraction) (0-20)
Variable 22. Joining-missing addend (subtraction) (0-99)
Variable 23. All other forms (0-20)
Variable 24. All other forms (0-99)

Open sentence objectives involve situations where a pupil is given an open sentence of the form a $\pm$ b = or $\pm \frac{a}{b}$ involving the numbers 0-10 (or 0-20), solves the sentence. For this category, two variables were specified.

Variable 25. Open addition sentences

Variable 26. Open subtraction sentences

Algorithm objectives refers to where a pupil is given two numbers whose sum is 0-99 and is to compute their sum; or where a pupil is given two numbers between 0 and 99 and is to compute their difference.

Variable 27. Addition algorithm

Variable 28. Subtraction algorithm

In addition to the coding of content a final variable,

Variable 29. Recommended time, was assigned to each part. The recommended amount of instructional time considered appropriate to spend on each part are necessarily approximate, but do reflect the relative importance given by the authors to particular parts, and to activities and to each topic.

Observations

Every day of instruction was observed and information was collected about what was being taught. This observation data was coded using "time-on-task" observational scheme which included a code about each part of each lesson which was being taught (Romberg, Small, Carnahan, & Cookson, 1979).[2] From these observations we then know the number of minutes spent on each part of an activity in each classroom.

In summary, to examine content coverage and instructional emphasis 29 variables were created. From these codes we are able to quantify how time is spent in classrooms on different aspects of the mathematics of addition and subtraction.

Results

The quantitative data about allocated time is summarized by classes within each grade. First, overall time allocated to each topic is presented, then the percent of recommended time spent on each part is used to summarize information

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[2]The observational procedure included data on teacher and student actions during instruction as well as content being taught.

on the content codes, and in turn these percents are used to characterize grades and classes.

Grade 1. The summary information on the number of parts taught, the number of minutes spent and the percent of recommended time spent for all eight classes in grade 1 is shown in Table 2. Similarly, the percent of recommended time to the 39 content codes is shown in Table 3.

To identify content emphasis both a "stem-and-leaf" diagram and a "box-and-whisker" diagram (Tukey, 1977) for the percent of recommended times on each content code for all the grade 1 classes is shown in Figure 1.

Finally, to examine between class differences the percent of recommended time for each class on the content codes was plotted with the average class recommended times. The plot for class 3 is shown in Figure 2.

Briefly, this data suggests that the eight grade 1 teachers were selective in what was taught. Overall they spent less time teaching mathematics than was recommended (particularly on topics S1 and S2). Table 3 shows there is considerable variability between classes on each code for percent of recommended time (e.g., DMP regular objectives 0 to 146%, other applications 8 to 114%, subtraction-comparison 0 to 116%). The average percent of recommended time spent on each code, illustrated in Figure 1 is a very tight distribution with most codes in the 50 to 70% range. The six "outliers" in the "box-and-whisker" diagram indicate that teaching toward the DMP regular objectives (and prerequisite/regular) were given more emphasis than other codes (94% and 72%). Also, it is interesting that subtraction-comparison sentences was also strongly emphasized (74%) since this was the primary approach to teach sentence writing in the commercial version of DMP and replaced with part-part-whole in these materials. Apparently, because of the teachers' prior experience, they spent more time on what they were familiar with than other sentences. At the other extreme they spent considerably less time on optional parts (little importance, 42%) and on parts that encouraged exploration (40%) or pupil discussion (39%).

In addition, each class was compared with their grade average profile. For five of the classes, although their total allocated times varied, the relative emphasis was very similar to the grade average profile. Two of the classes differed significantly but were similar to each other. In these classes (see Figure 2 for classroom 3) considerably more emphasis was given to

Table 2

Number of Parts Taught, Allocated Time and Percent of Recommended Time by Topic and Class for Grade 1

	S-1			S-2			S-3			Total		
	N Parts (13)	Minutes	%	N Parts (15)	Minutes	%	N Parts (12)	Minutes	%	N Parts (40)	Minutes	%
Class												
1	11	372	74	13	329	49	11	411	81	35	1112	66
2	10	238	47	10	252	38	8	160	32	28	650	39
3	10	308	61	12	416	63	10	505	100	32	1229	73
4	0	0	0[a]	11	396	60	11	452	90	22	848	51
5	8	386	76	6	186	28[b]	0	0	0[b]	14	572	34
6	10	300	59	11	212	32	6	130	26	27	642	38
7	11	398	79	12	392	59	9	509	101	32	1299	78
8	13	427	85	12	412	62	12	599	119	37	1438	86
Average	10[c]	304[c]	69[c]	11	324	49	10[c]	346[c]	78[c]			

[a]The observation data were not usable due to inadequate coding.

[b]The teacher used only portions of S-2 and none of S-3.

[c]The average is based on 7 classes.

Table 3

Percent of Recommended Time by Content Goals and Instructional Objectives and by Class for All Topics for Grade 1

	Classification of DMF Objectives				Importance of Objectives			Algorithmic Nature of Teaching			Intended Outcomes					Applications					
	Prereq.	Rep.	Rev.	Pre./Rep.	Great	Mod.	Little	Alg.	Non-Alg.	Alg./ Non-Alg.	Skill Prep.	Skill Acquis.	Skill Maint.	Concreteness	Diversity of Approach	Other Appli.	Stand. Appli.	No Appli.	Explorability	Encouragement of Pupil Discussion	Cohesion
Class																					
1	60	96	0	76	66	69	62	76	63	0	23	60	75	57	63	74	55	83	68	53	55
2	42	31	0	32	43	38	24	37	40	0	0	44	35	34	36	40	37	40	28	24	30
3	62	146	0	82	79	95	16	93	65	0	120	60	85	55	71	68	61	102	37	42	68
4	34	109	0	82	53	57	31	78	39	0	60	45	56	32	34	46	35	86	26	21	64
5	49	0	0	0	29	44	35	19	40	0	0	34	36	26	23	8	51	21	24	23	17
6	44	21	0	28	38	37	42	21	46	0	23	37	40	37	37	49	42	23	26	33	32
7	68	131	0	88	87	67	62	78	77	0	40	83	74	58	71	83	71	86	48	54	75
8	72	124	0	117	94	81	64	69	93	0	83	78	94	70	90	114	81	73	67	64	75
Ave.	54	94	0	72	61	61	42	59	58	0	58	55	62	46	53	60	54	64	40	39	52
Total Recommended Minutes	1170	140	0	365	960	460	255	490	1185	0	30	825	820	990	715	345	885	445	380	920	390

Table 3 (continued)

	Sentence-writing														Open Sentences		Algorithms	
	Add-Simple Joining 0-20	Add-Simple Joining 0-99	Subt-Simple Separating 0-20	Subt-Simple Separating 0-99	Subt-Part Part Whole 0-20	Subt-Part Part Whole 0-99	Add-Part Part Whole 0-20	Add-Part Part Whole 0-99	Subt-Comparison 0-20	Subt-Comparison 0-99	Subt-Join-Addend 0-20	Subt-Join-Addend 0-99	Other 0-20	Other 0-99	Addition 0-20	Subtraction 0-20	Addition	Subtraction
Class																		
1	75	0	77	0	50	0	46	0	73	0	0	0	0	0	77	76	0	0
2	47	0	49	0	26	0	28	0	40	0	0	0	0	0	40	37	0	0
3	76	0	73	0	61	0	57	0	96	0	0	0	0	0	93	92	0	0
4	32	0	26	0	71	0	71	0	67	0	0	0	0	0	78	78	0	0
5	63	0	67	0	9	0	23	0	0	0	0	0	0	0	21	19	0	0
6	56	0	60	0	39	0	40	0	46	0	0	0	0	0	22	21	0	0
7	85	0	85	0	89	0	80	0	79	0	0	0	0	0	84	78	0	0
8	100	0	100	0	105	0	96	0	116	0	0	0	0	0	68	69	0	0
Ave.	67	0	67	0	56	0	55	0	74	0	0	0	0	0	60	59	0	0
Total Recommended Minutes	710	0	670	0	330	0	400	0	280	0	0	0	0	0	455	490	0	0

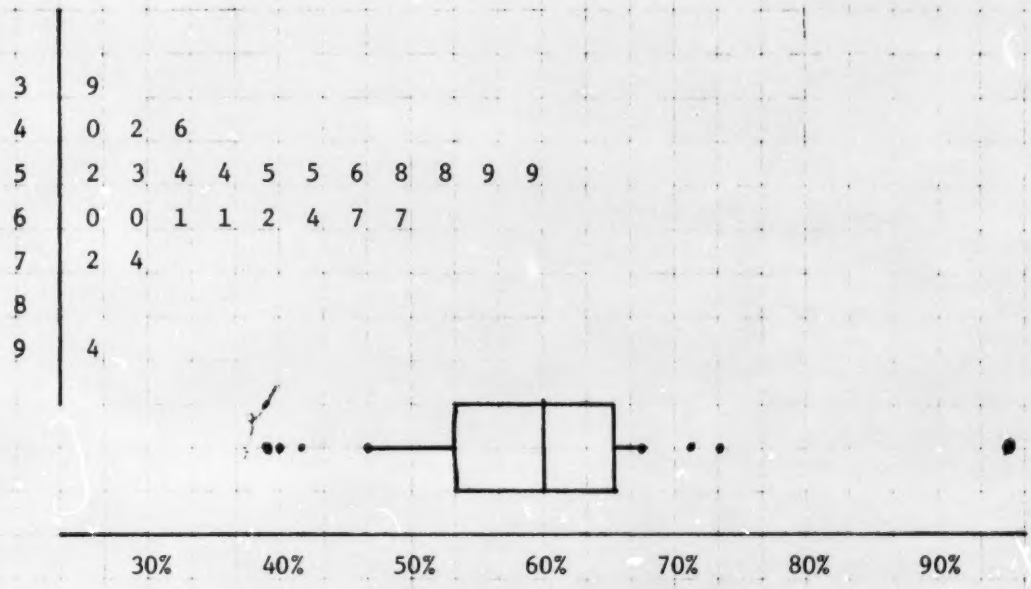
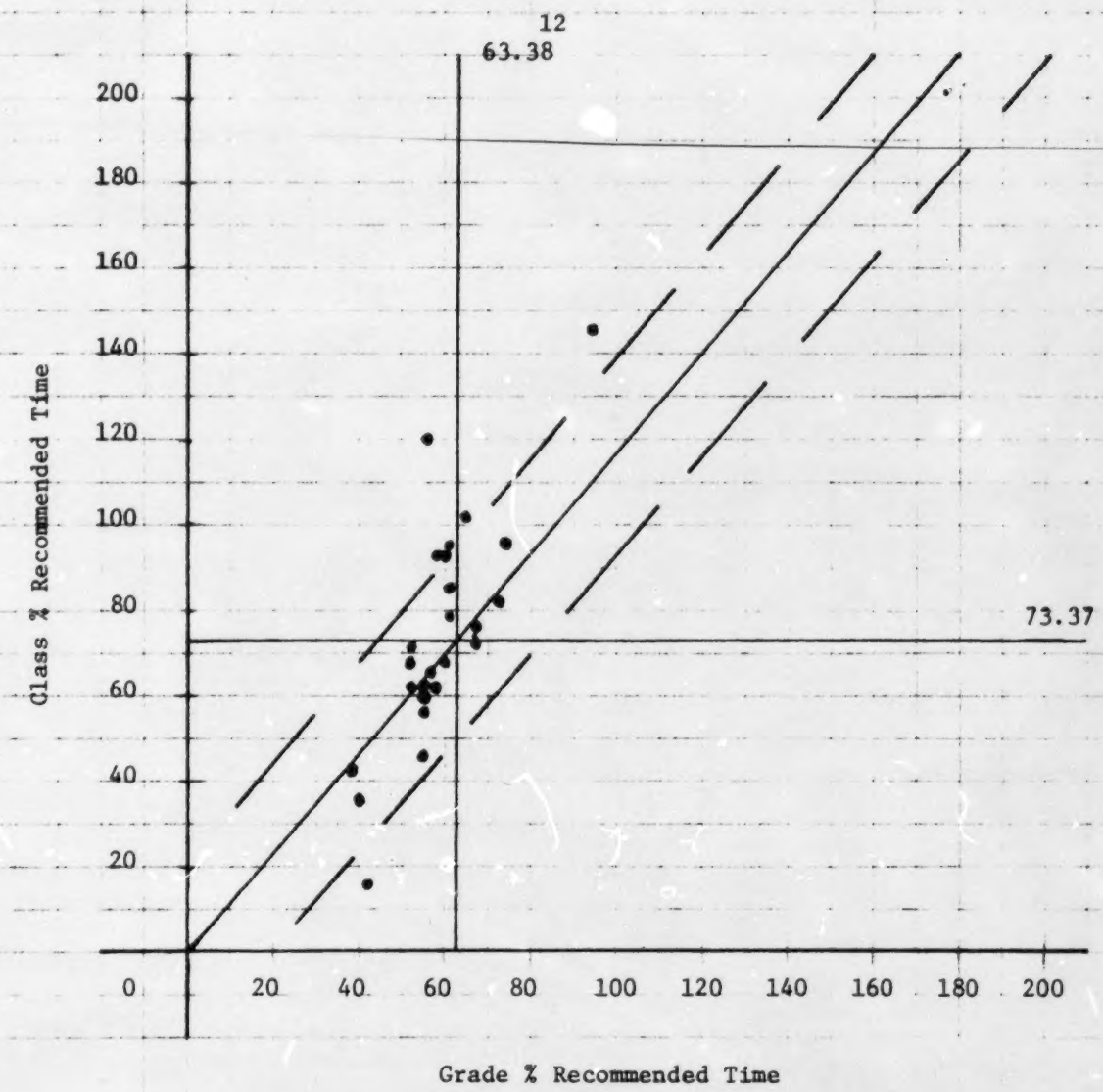


Figure 1. "Stem-and-leaf" and "box-and-whiskers" diagrams of average percent of recommended time for all codes on all topics for grade 1.



Above %	Code	Below %	Code
120	Skill Preparation	16	Objective Little Importance
146	DMP Regular Objective		
102	No Applications		
95	Objective Moderate Importance		
93	Algorithmic Teaching		
93	Open Sentences--Addition		
92	Open Sentences--Subtraction		

Figure 2. Plot of class and grade percent recommended tmes for all topics: Grade 1, class 3.

skill development. The data for the final class failed to fit either category because much of the curriculum was not taught (Class 5). In fact, this teacher felt DMP was not structured enough and replaced DMP with math worksheets.

Overall, this data suggests that in these classrooms the DMP activities were modified in systematic ways both to lessen the children's opportunity to explore and discuss mathematics and to emphasize acquiring skills.

Grade 2. Because the topics taught in grade 2 involved both sentence writing (S) and algorithmic (A) topics, the data for the six classes in this grade have been summarized for both content aspects. Overall time allocated to each topic is shown in Table 4. The percents of recommended times to each content code for the S topics is reported in Table 5 and for the A topics in Table 6.

The "stem-and-leaf" and "box-and-whisker" diagrams for the class average percents of recommended times on each code are shown in Figure 3. And finally the plots comparing class and grade recommended times for class 4 on the S topics and class 5 on the A topics are in Figures 4 and 5.

In grade 2 considerable time was spent on the initial topic (S4) (probably to get a "good" start and to review ideas from grade 1). Topics S5 and A1 were reasonably covered; Topic S6 was barely taught (an average of 9 of 26 parts taught). This topic is filled with activities to reinforce mastery of sentence writing and addition and subtraction facts. Apparently, these teachers felt the prior work in S4 and S5 were sufficient. Topic A2 was reasonably covered in five of the six classes but quickly because it was the end of the school year.

The percent of recommended times on content codes and the average recommended times for all six classes on the S Topics show considerable variability across codes but less between class variability than was evident in grade 1. The "stem-and'leaf" diagram for the S Topics shows a bimodal distribution with heavy emphasis on all sentence writing objectives, and little use of optional activities. For the A Topics more between class variability is evident and the distribution of percents is much tighter. The outliers reflect more overall emphasis on open-sentences, little use of optional activities, cohesive activities, or skill preparation. Furthermore, similarity across the classes is evident in that for four of the six classes the pattern of emphasis was similar to the average for both the S and A Topics. Only class 4 on the S Topics and class 5 on the A Topics varied. The teacher in class 4 (see Figure 4), who

Table 4

Number of Parts Taught, Allocated Time and Percent of Recommended Time by Topic and Class for Grade 2

	S-4			S-5			S-6			A-1			A-2			Total		
	N Parts (12)	Minutes	%	N Parts (15)	Minutes	%	N Parts (26)	Minutes	%	N Parts (13)	Minutes	%	N Parts (14)	Minutes	%	N Parts (80)	Minutes	%
Class																		
1	12	654	105	14	420	68	10	280	34	12	485	99	14	401	73	62	2240	72
2	11	908	145	11	485	79	7	347	43	11	420	86	13	437	79	53	2597	84
3	12	858	137	14	620	101	10	430	53	11	598	122	13	523	95	60	3029	98
4	11	909	145	11	713	116	7	503	62	10	487	99	5	234	43[a]	44	2846	92
5	12	600	96	12	571	93	10	363	45	13	577	118	13	410	75	60	2521	81
6	10	820	131	9	406	66	9	347	43	11	424	87	11	517	94	50	2514	81
Average	11	791	127	12	536	87	9	378	46	11	499	102	11	420	76			

[a]This class had completed only half of the topic when the school year ended.

Table 5

Percent of Recommended Time by Content Goals and Instructional Objectives and by Class for All Topics for Grade 2S

	Classification of DMP Objectives				Importance of Objectives			Algorithmic Nature of Teaching			Intended Outcomes					Applications					
	Prereq.	Reg.	Rev.	Pre./Reg.	Great	Mod.	Little	Alg.	Non-Alg.	Alg./ Non-Alg.	Skill Prep.	Skill Acquis.	Skill Maint.	Concreteness	Diversity of Approach	Other Appl.	Stand. Appl.	No Appl.	Explorability	Encouragement of Pupil Discussion	Cohesion
Class																					
1	87	34	0	0	116	67	14	44	84	94	0	86	60	67	54	71	95	49	58	57	83
2	112	43	0	0	165	82	6	58	103	133	0	92	82	62	64	91	124	63	69	57	98
3	119	53	0	0	142	105	20	63	117	114	0	119	85	97	85	113	127	68	80	91	125
4	131	62	0	0	181	108	16	42	153	185	0	136	93	121	96	139	176	53	57	84	138
5	94	45	0	0	117	82	18	51	87	130	0	115	62	86	62	73	110	58	62	70	102
6	99	43	0	0	121	89	9	53	95	110	0	90	73	56	62	74	114	59	61	64	99
Ave.	107	46	0	0	140	89	14	52	107	131	0	106	76	82	70	94	124	59	64	70	107
Total Recommended Minutes	1240	815	0	0	550	965	540	985	850	220	0	480	1575	455	800	440	530	1085	850	840	520

Table 5 (continued)

	Sentence-writing														Open Sentences		Algorithms	
	Add-Simple Joining 0-20	Add-Simple Joining 0-99	Subt-Simple Separating 0-20	Subt-Simple Separating 0-99	Subt-Part Part Whole 0-20	Subt-Part Part Whole 0-99	Add-Part Part Whole 0-20	Add-Part Part Whole 0-99	Subt-Comparison 0-20	Subt-Comparison 0-99	Subt-Join-Addend 0-20	Subt-Join-Addend 0-99	Other 0-20	Other 0-99	Addition 0-20	Subtraction 0-20	Addition	Subtraction
Class																		
1	109	0	104	0	108	0	105	0	70	0	116	0	104	0	52	53	0	0
2	144	0	135	0	155	0	153	0	85	0	164	0	139	0	66	68	0	0
3	141	0	144	0	140	0	149	0	103	0	146	0	149	0	73	75	0	0
4	204	0	214	0	225	0	231	0	116	0	218	0	195	0	65	67	0	0
5	114	0	122	0	105	0	101	0	71	0	108	0	114	0	67	68	0	0
6	129	0	129	0	140	0	141	0	72	0	138	0	119	0	61	63	0	0
Ave.	140	0	141	0	145	0	147	0	86	0	149	0	137	0	64	66	0	0
Total Recommended Minutes	5520	0	355	0	475	0	410	0	435	0	325	0	350	0	1245	1205	0	0

Table 6

Percent of Recommended Time by Content Goals and Instructional Objectives and by Class for All Topics for Grade 2A

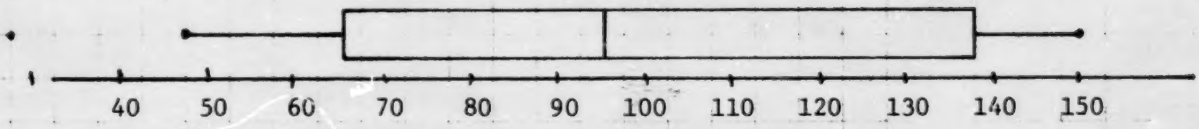
	Classification of DMP Objectives				Importance of Objectives			Algorithmic Nature of Teaching			Intended Outcomes					Applications					
	Prereq.	Reg.	Rev.	Pre./Reg.	Great	Mod.	Little	Alg.	Non-Alg.	Alg./ Non-Alg.	Skill Pref.	Skill Acquis.	Skill Maint.	Concreteness	Diversity of Approach	Other Appl.	Stand. Appl.	No Appl.	Explorability	Encouragement of Pupil Discussion	Cohesion
Class																					
1	85	0	0	0	86	91	0	82	93	90	35	83	93	73	82	105	77	84	95	78	44
2	82	0	0	0	86	81	0	77	54	131	25	93	73	81	100	93	98	75	45	72	62
3	108	0	0	0	115	98	37	107	109	110	43	117	102	109	102	105	133	100	73	83	38
4	69	0	0	0	80	50	7	74	43	76	75	84	49	84	44	29	91	70	46	70	0
5	95	0	0	0	91	106	63	75	140	123	25	91	107	89	118	156	109	78	106	94	77
6	90	0	0	0	88	105	3	93	56	116	0	96	92	81	95	86	87	93	43	62	26
Ave.	88	0	0	0	91	89	28	85	83	108	41	94	86	86	90	96	99	83	68	77	50
Total Recommended Minutes	1040	0	0	0	715	295	30	675	180	185	40	580	420	550	350	130	230	680	275	635	115

Table 6 (continued)

	Sentence-writing														Open Sentences		Algorithms	
	Add-Simple Joining 0-20	Add-Simple Joining 0-99	Subt-Simple Separating 0-20	Subt-Simple Separating 0-99	Subt-Part Part Whole 0-20	Subt-Part Part Whole 0-99	Add-Part Part Whole 0-20	Add-Part Part Whole 0-99	Subt-Comparison 0-20	Subt-Comparison 0-99	Subt-Join-Addend 0-20	Subt-Join-Addend 0-99	Other 0-20	Other 0-99	Addition 0-20	Subtraction 0-20	Addition	Subtraction
Class																		
1	0	89	0	86	0	99	0	98	0	99	0	69	0	84	150	150	81	102
2	0	82	0	93	0	99	0	98	0	99	0	139	0	110	140	140	89	97
3	0	140	0	77	0	124	0	150	0	124	0	168	0	131	175	175	92	116
4	0	78	0	61	0	89	0	75	0	89	0	127	0	79	205	205	55	98
5	0	107	0	124	0	167	0	132	0	167	0	129	0	137	175	175	87	121
6	0	98	0	77	0	94	0	111	0	94	0	128	0	93	170	170	95	102
Ave.	0	99	0	86	0	112	0	111	0	112	0	127	0	106	169	169	83	106
Total Recommended Minutes	0	340	0	140	0	150	0	290	0	150	0	75	0	205	20	20	750	370

1	4
2	
3	
4	6
5	2 9
6	4 4 6
7	0 0 6
8	2 6 9
9	4
10	6 7 7 7
11	
12	4
13	1 7
14	0 0 1 5 7 9

Grade 2 S Topics



2	8
3	
4	1
5	0
6	8
7	7
8	3 3 3 5 6 6 6 8 9
9	0 1 4 6 9 9
10	6 8 8
11	1 2 2
12	7
13	
14	
15	
16	9 9

Grade 2 A Topics

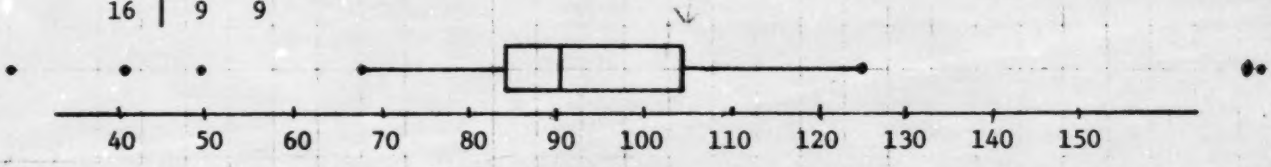
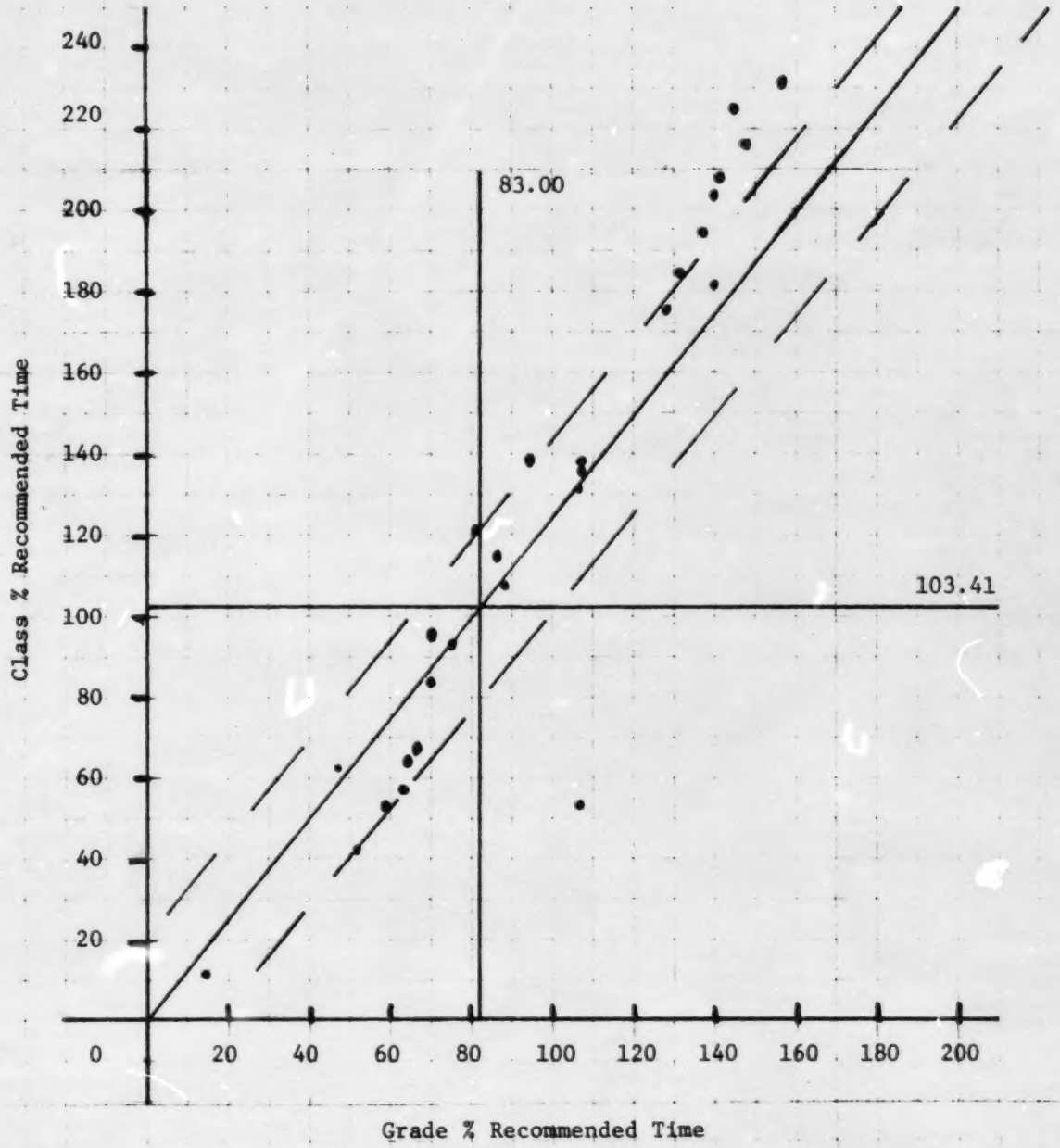


Figure 3. "Stem-and-leaf" and "box-and-whisker" diagrams of average percent of recommended time for all codes on the S topics and the A topics for grade 2.



Above		Below	
%	Code	%	Code
195	Other sentences 0-20	57	Explorability
204	Add-simple join 0-20		
214	Subtract-simple separate 0-20		
218	Subtract-join addend 0-20		
225	Subtract-part/part/whole 0-20		
231	Add-part/part/whole 0-20		

Figure 4. Plot of class and grade percent recommended times for all codes on the S topics: Grade 2, class 4.

believed the students were the "low" group spent considerable more time on six sentence writing objectives (on the A Topics this class was not different). Class 5, on other other hand (see Figure 5), who had the "high" group, did not differ from the average profile on the S Topics but on the A Topics spent considerably more time allowing students to do optional activities, explore problems, work on other applications, etc.

In summary, the modification of the instructional materials toward a skills orientation by teachers in grade 2 is as evident here as in grade 1. The only new element is the belief that children with different levels of "ability" should do different things; more "drill and practice" for "low" students, and more "explorations" for "good" students.

Grade 3. The same summary tables and figures for the six grade 3 classes are reported in Tables 7 and 8 and Figures 6 and 7.

For these six classes the teaching of these topics when the two-digit algorithms are to be mastered is important. Overall, more time is spent on these topics than was recommended. However, across the content codes there is considerable variability. The distribution of average recommended time is tight (97 to 128%). Only work on addition and subtraction facts and worksheets are emphasized and optional activities are not used (not at all in four classes).

Finally, five of the six classes are very similar to the average profile for grade 3 (see Figure 7). Only class 3 differs. Here, with "good" students optional activities are taught with lots of work on concrete applications, and algorithmic performance is emphasized less.

Summary

Each of the 20 classes observed in this study differs from the others on the dimension of what mathematics content is included and what is emphasized when teaching children to add and subtract. The quantitative data on how much time was spent on various aspects of mathematics, however, reveals four important features:

First, the primary difference between classes at each grade level was in terms of total allocated time to mathematics and not on what was emphasized.

Second, teachers modified the program by selecting parts to be taught and how much time was then spent on each part. Furthermore, the dominant pattern was not to select activities which encourage discussion and exploration and to

Table 7

Number of Parts Taught, Allocated Time and Percent of Recommended Time by Topic and Class for Grade 3

	A-3			A-4			Total		
	N Parts (13)	Minutes	%	N Parts (15)	Minutes	%	N Parts (28)	Minutes	%
Class									
1	13	696	129	7	377	122	20	1073	126
2	12	620	115	6	374	121	18	994	117
3	12	743	138	7	327	105	19	1070	126
4	12	600	111	6	301	97	18	901	106
5	12	510	94	6	252	81	18	762	90
6	12	705	131	6	465	150	18	1170	138
Average	12	646	120	6	349	113			

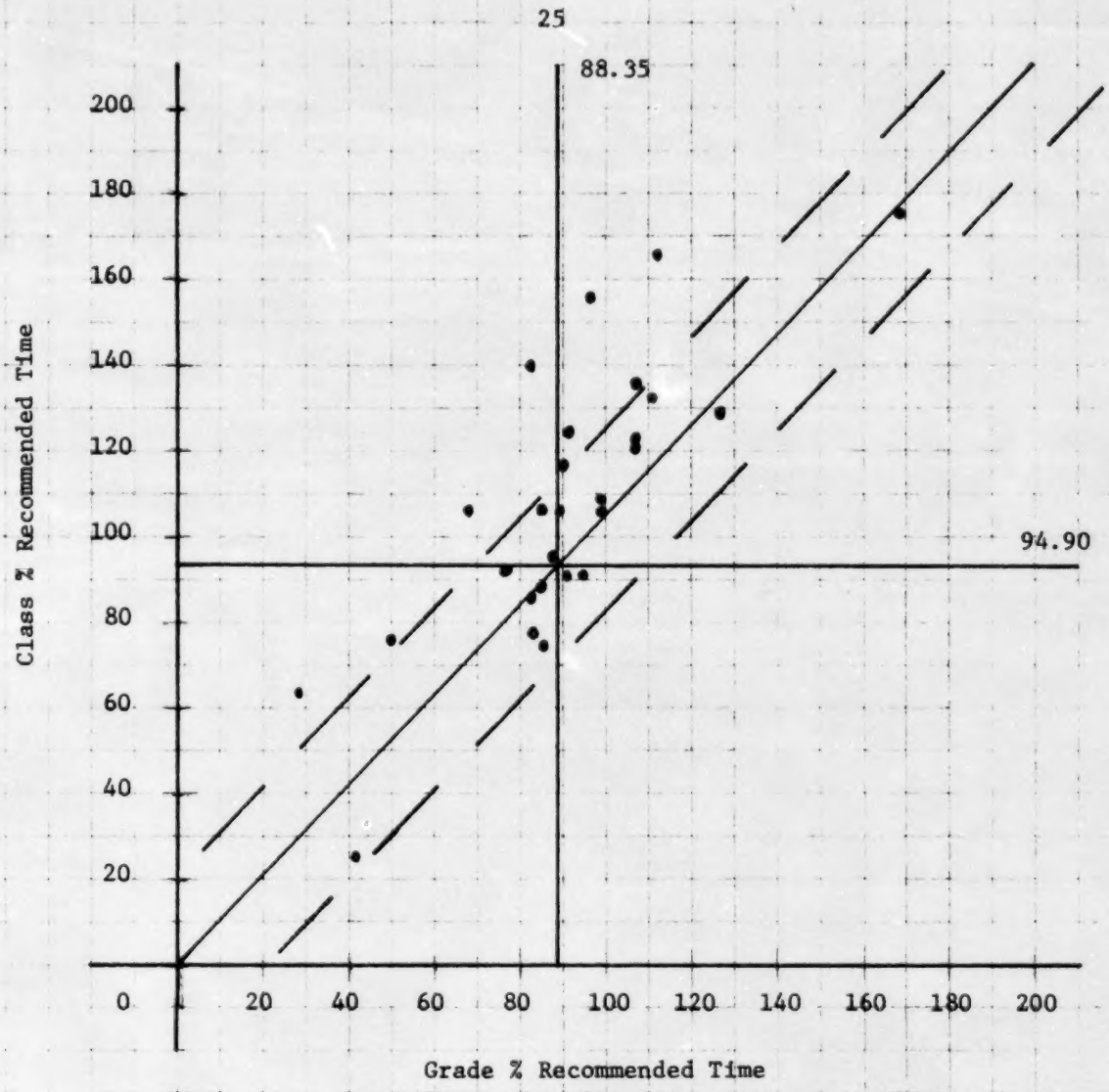
Table 8 (continued)

	Sentence-writing														Open Sentences		Algorithms	
	Add-Simple Joining 0-20	Add-Simple Joining 0-99	Subt-Simple Separating 0-20	Subt-Simple Separating 0-99	Subt-Part Part Whole 0-20	Subt-Part Part Whole 0-99	Add-Part Part Whole 0-20	Add-Part Part Whole 0-99	Subt-Comparison 0-20	Subt-Comparison 0-99	Subt-Join-Addend 0-20	Subt-Join-Addend 0-99	Other 0-20	Other 0-99	Addition 0-20	Subtraction 0-20	Addition	Subtraction
Class																		
1	0	114	0	111	0	134	0	140	0	110	0	151	0	105	163	163	151	113
2	0	99	0	123	0	128	0	133	0	98	0	156	0	100	110	110	114	95
3	0	167	0	115	0	126	0	133	0	141	0	128	0	103	143	143	91	115
4	0	86	0	99	0	98	0	106	0	98	0	113	0	102	177	177	102	88
5	0	71	0	75	0	66	0	73	0	72	0	79	0	63	123	123	98	90
6	0	120	0	132	0	113	0	120	0	123	0	140	0	105	160	160	174	145
Ave.	0	109	0	109	0	111	0	117	0	107	0	128	0	97	146	146	122	108
	0	285	0	425	0	275	0	240	0	415	0	190	0	315	30	30	250	440

Table

Percent of Recommended Time by Content Goals and Instructional Objectives and by Class for All Topics for Grade 3

	Classification of DMP Objectives				Importance of Objectives			Algorithmic Nature of Teaching			Intended Outcomes					Applications					
	Prereq.	Req.	Rev.	Pre./Req.	Great	Mod.	Little	Alg.	Non-Alg.	Alg./ Non-Alg.	Skill Prep.	Skill Acquis.	Skill Maint.	Concreteness	Diversity of Approach	Other Appli.	Stand. Appli.	No Appli.	Explorability	Encouragement / Pupil Discussic	Cohesion
Class																					
1	129	122	0	0	119	145	109	127	116	141	0	84	145	101	146	107	128	138	133	125	197
2	115	121	0	0	118	130	0	119	111	124	0	96	126	110	145	108	89	158	111	100	69
3	138	105	0	0	109	167	103	116	150	97	0	99	137	163	147	191	102	106	128	129	103
4	111	97	0	0	106	120	0	101	102	117	0	96	110	93	113	109	97	115	98	77	99
5	94	81	0	0	77	132	0	102	71	107	0	80	94	86	106	87	64	123	104	89	72
6	131	150	0	0	130	176	0	172	121	133	0	130	141	163	154	169	104	156	149	147	119
Ave.	120	113	0	0	110	145	35	123	112	120	0	98	125	119	135	128	97	133	121	111	110
	540	310	0	0	570	245	35	220	385	245	0	255	595	290	440	215	350	285	430	340	75



Above		Below	
%	Code	%	Code
63	Little Importance		
106	Explorability		
124	Subtract-Simple Separate 0-99		
140	Non-Algorithmic		
156	Other Applications		
167	Subtract-part/part/whole 0-99		

Figure 5. Plot of class and grade percent recommended times for all codes on the A topics: Grade 2, class 5.

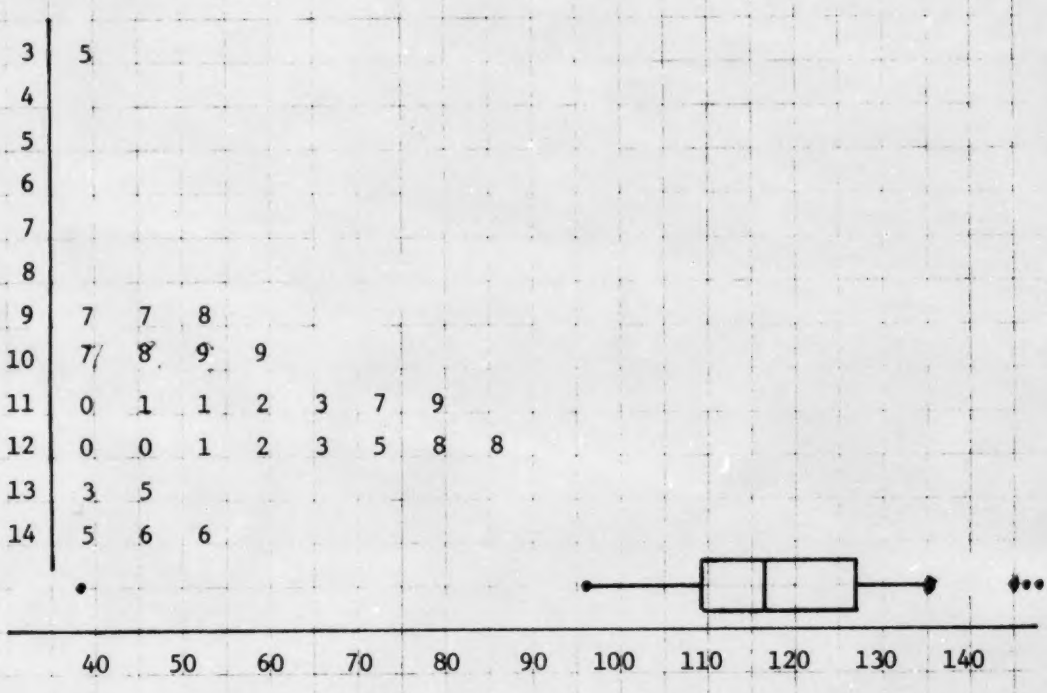
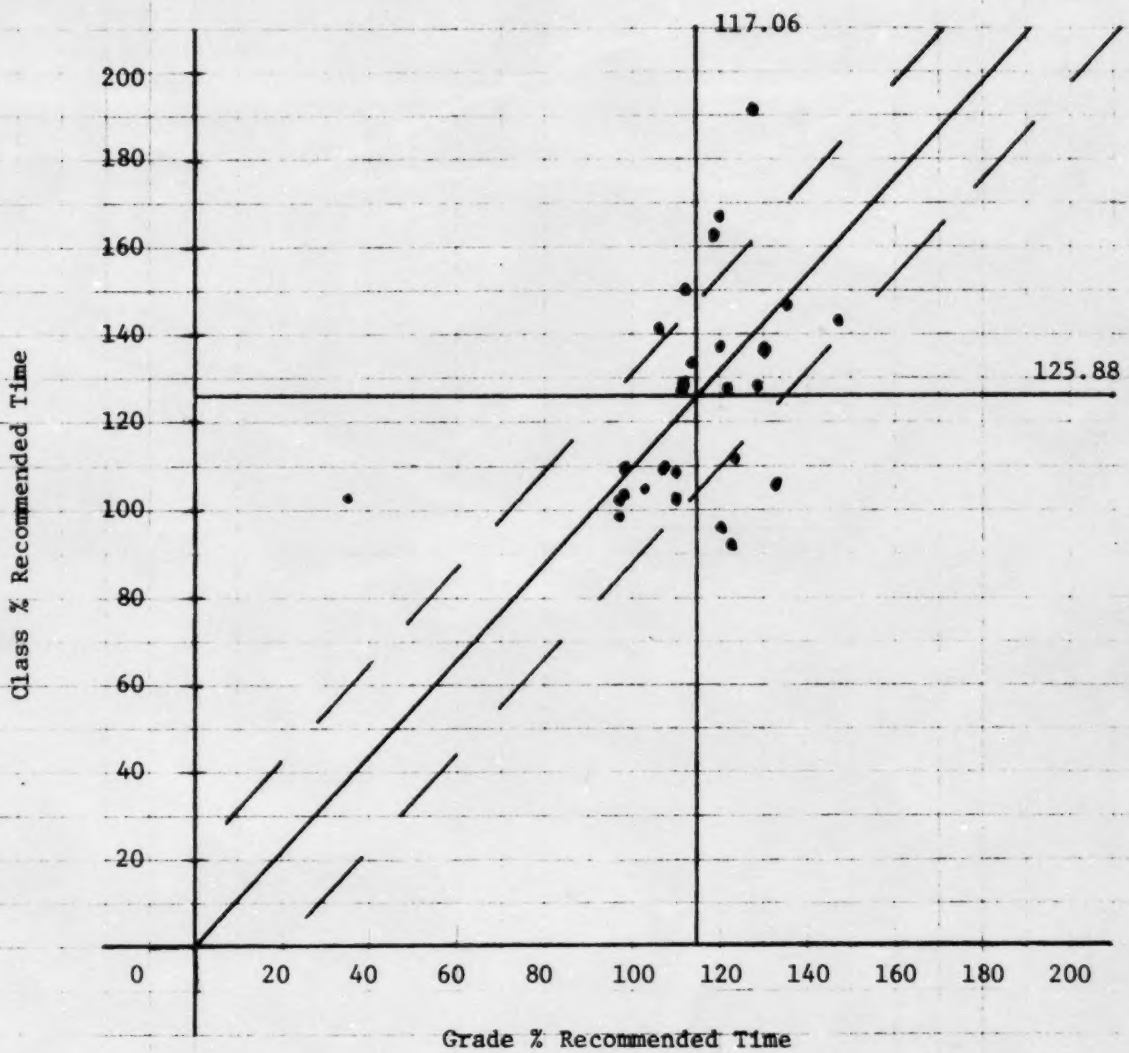


Figure 6. "Stem-and-leaf" and "box-and-whisker" diagrams of average percent of recommended times for all codes on all topics for grade 3.



Above		Below	
%	Code	%	Code
103	Little Importance	91	Addition Algorithm
140	Subtraction-Comparison 0-99	97	Algorithm/Non-Algrithm Teaching
150	Non-algorithmic	106	No Application
163	Concreteness		
167	Add-simple join 0-99		
191	Other Applciations		

Figure 7. Plot of class and grade percent recommended times for all topics: Grade 3, class 3.

emphasize skill development and practice via worksheets.

Third, for a class of "poor" students the dominant pattern of activities was accentuated. These students were given more practice on skills and less opportunity to explore and discuss ideas.

Finally, only with classes of "good" students were there any variations. With these students while practice of skills was not diminished, more optional activities were selected and more time was spent on exploration.

In conclusion, the mathematics program that was actually taught in these classrooms varied considerably from the intentions of the developers. For children, what it means to know and do mathematics is effected by the deliberate selection of activities and how they are emphasized in classrooms.

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